Signatures of modified gravity on the CMB

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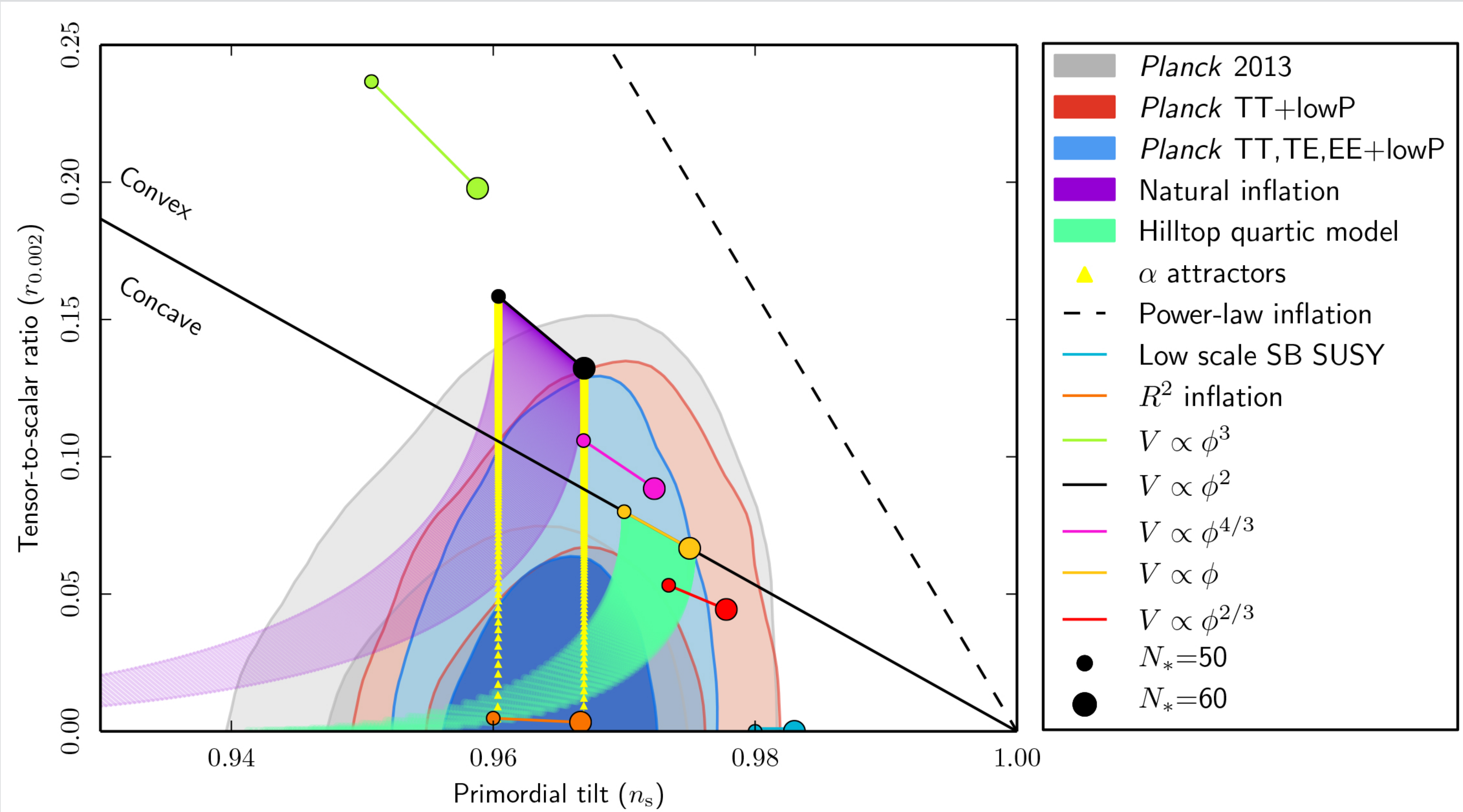
Summary

- B-modes
- GW at recombination
- Modified speed
- Graviton mass
- Bigravity
- Instabilities
- Summary

Introduction

- Gravitational waves during inflation
- Carry information about the energy of inflation
- Detectable on the B-modes



Planck 2015

We know a lot about inflation and constraints are expected to get better on the next years

B modes

- Quadruple distribution for the Thompson scattering
- Source by the gravitational waves background

source

$$C^T_{BB,l} = (4\pi)^2 \int k^2 \mathrm{d}k P_h(k) \left| \int \mathrm{d}\tau g(\tau) \Psi(k,\tau) \left[2j_l' + 4\frac{j_l}{x} \right] \right|^2$$

Initial conditions

projection

B modes

- Large scales, $\Psi \propto \dot{h}(\tau_{\rm rec}) e^{-(k\Delta\tau_{\rm rec})^2/2}$
- Small scales neutrino damping and other processes take place

GW background

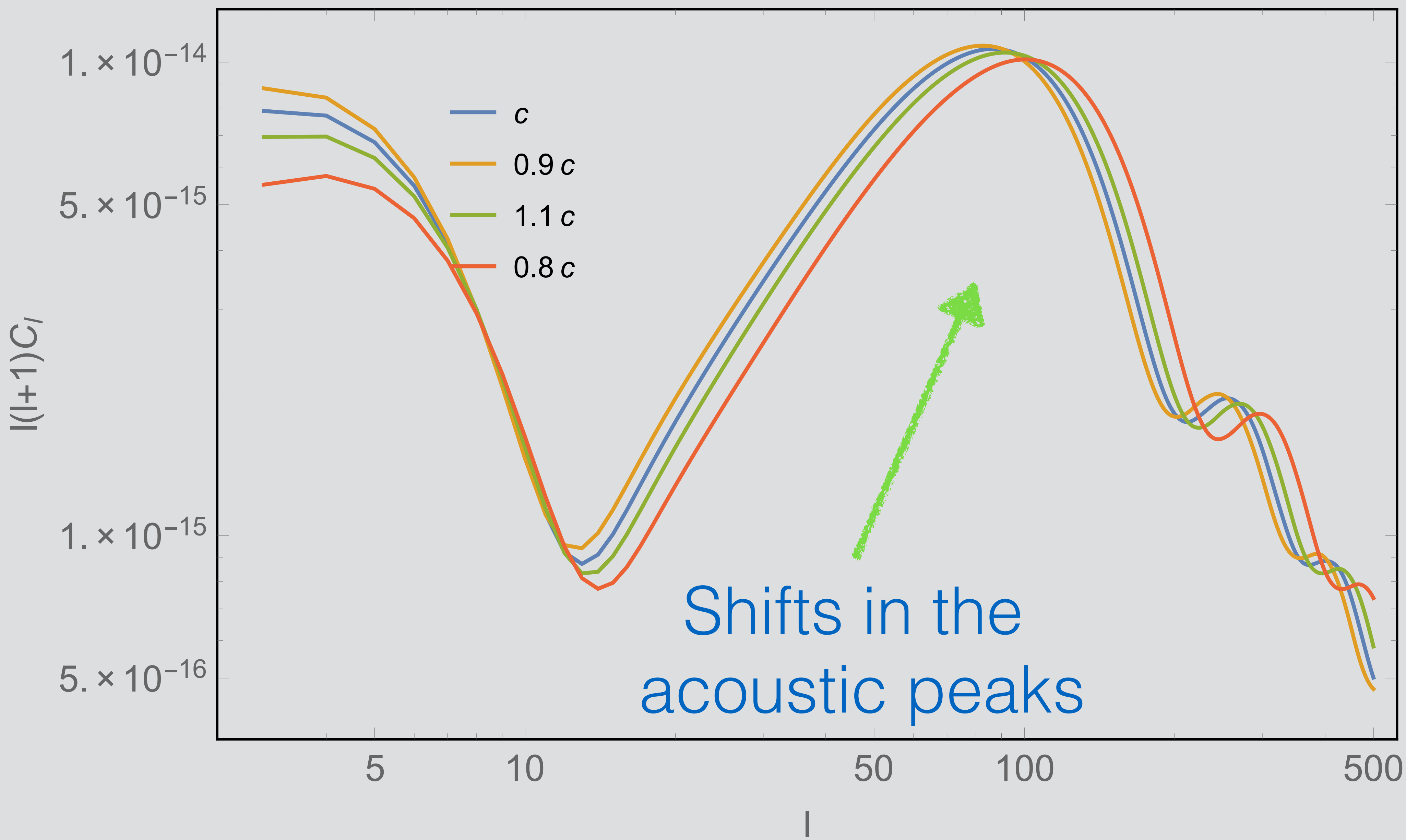
- Matter domination
  - Important at large scales. Out of the horizon for most of the time
- Radiation domination
  - More important at late times. Gravitational waves not so sensitive to changes into this because at recombination most of the are already out of the horizon

Modified gravity

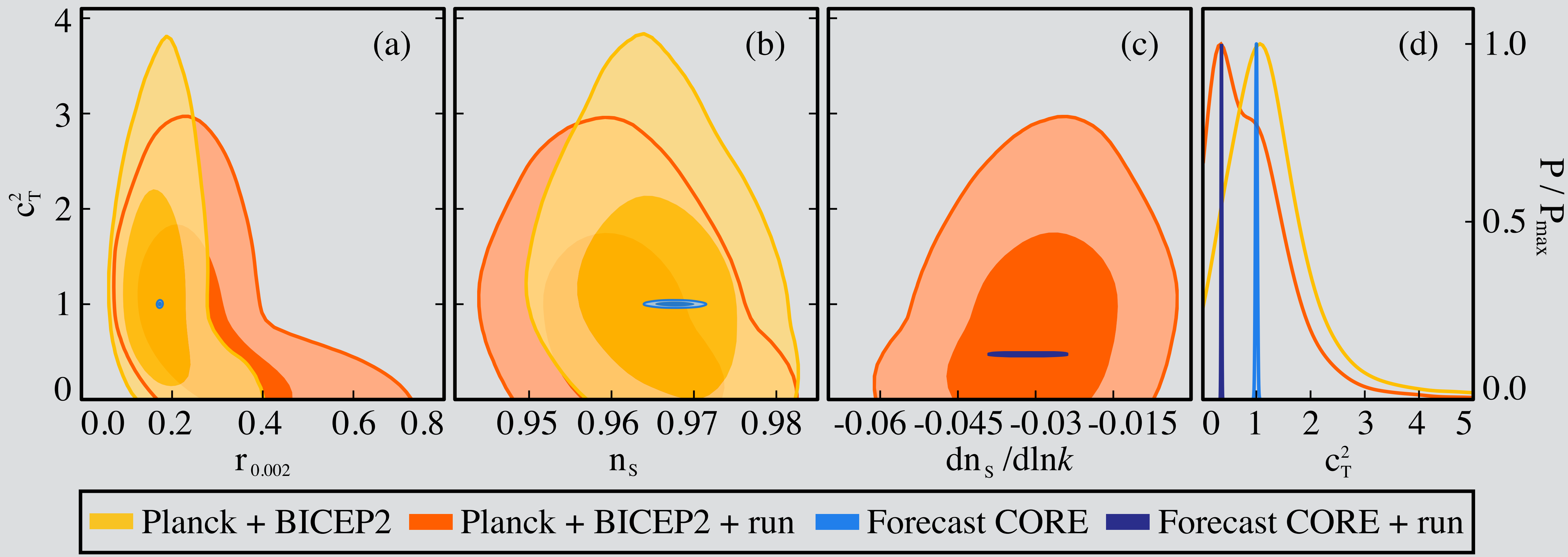
- Which signatures can we probe by this.
- On large scales, a modification of the speed and a mass for the gravitational field could have had important signatures
- We will focus on modified tensor speed and graviton mass

Modified tensor speed

- Occurs in all Horndenski type modifications.
- Changes the horizon so fields enter at a different time.
- It modifies the acoustic peaks structure.
- Changes directly proportional to the c_T/c



$1. \times 10^{-14}$
$5. \times 10^{-15}$
$1. \times 10^{-15}$
$5. \times 10^{-16}$
$l(l+1)C_l$
c
$0.9\,c$
$1.1\,c$
$0.8\,c$
5
10
50
100
500
l
Shifts in the
acoustic peaks



Rivera et al. (2014)

Modified tensor speed

- Changes to speed are to be considered as a plausible modification

- Disformal transformation does not set it to 1, but it can be useful to understand what are the most important modifications.

Barrage, SC and Davis (2016)

Massive graviton

- Now let's consider a massive graviton under a cosmological background

$$h'' + \left(k^2 + m^2 a^2 - \frac{a''}{a}\right) h = 0$$

- We need to consider matter domination first.

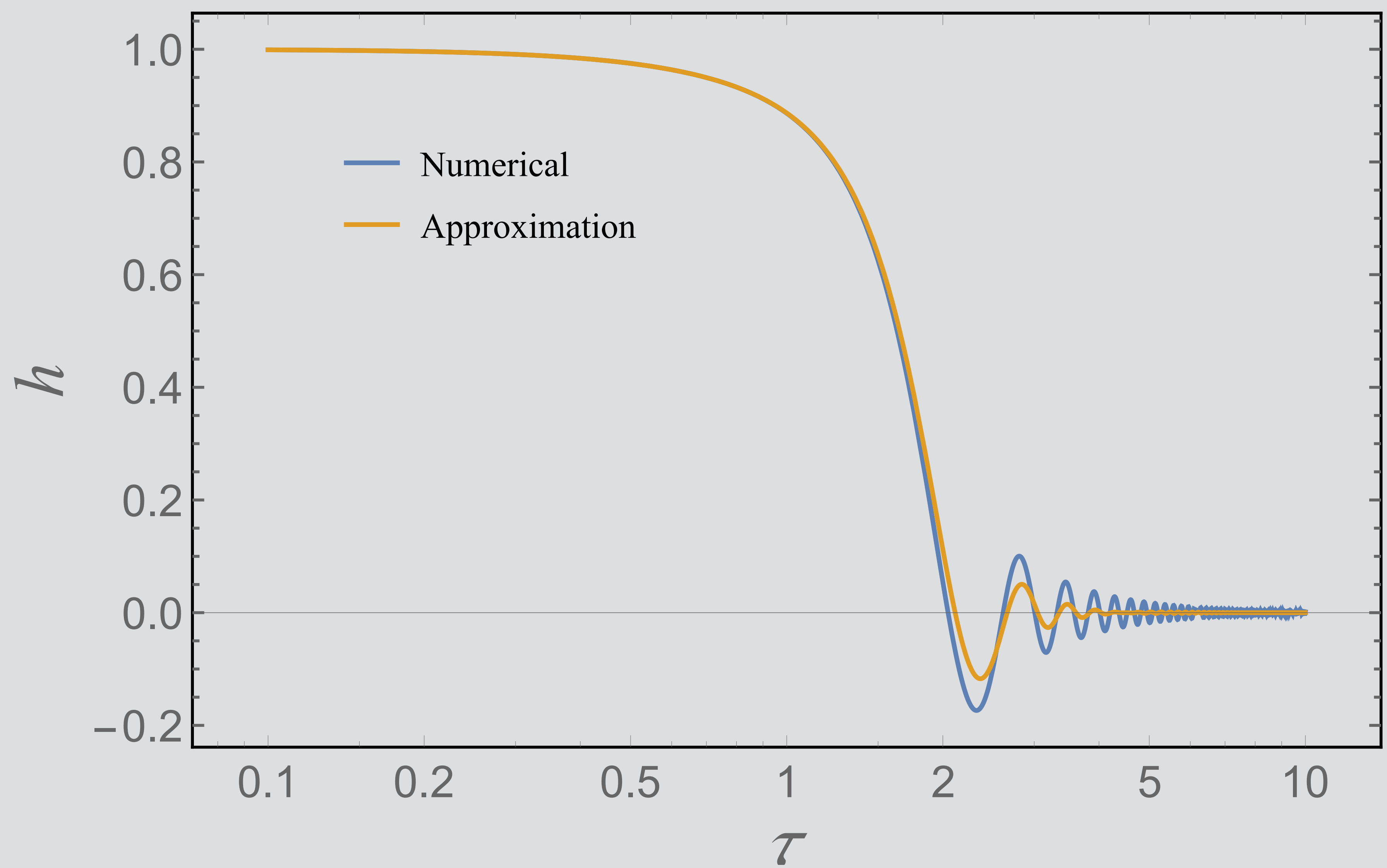
$$a \propto \tau^2$$

- Approximated solution by,

$$h = 3\frac{j_1(k\tau)}{k\tau} \times j_0(\frac{1}{3}mH_0^2\tau^3)$$

Massless part

Massive part


Numerical
Approximation
h
τ
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0.1
0.2
0.5
1
2
5
10

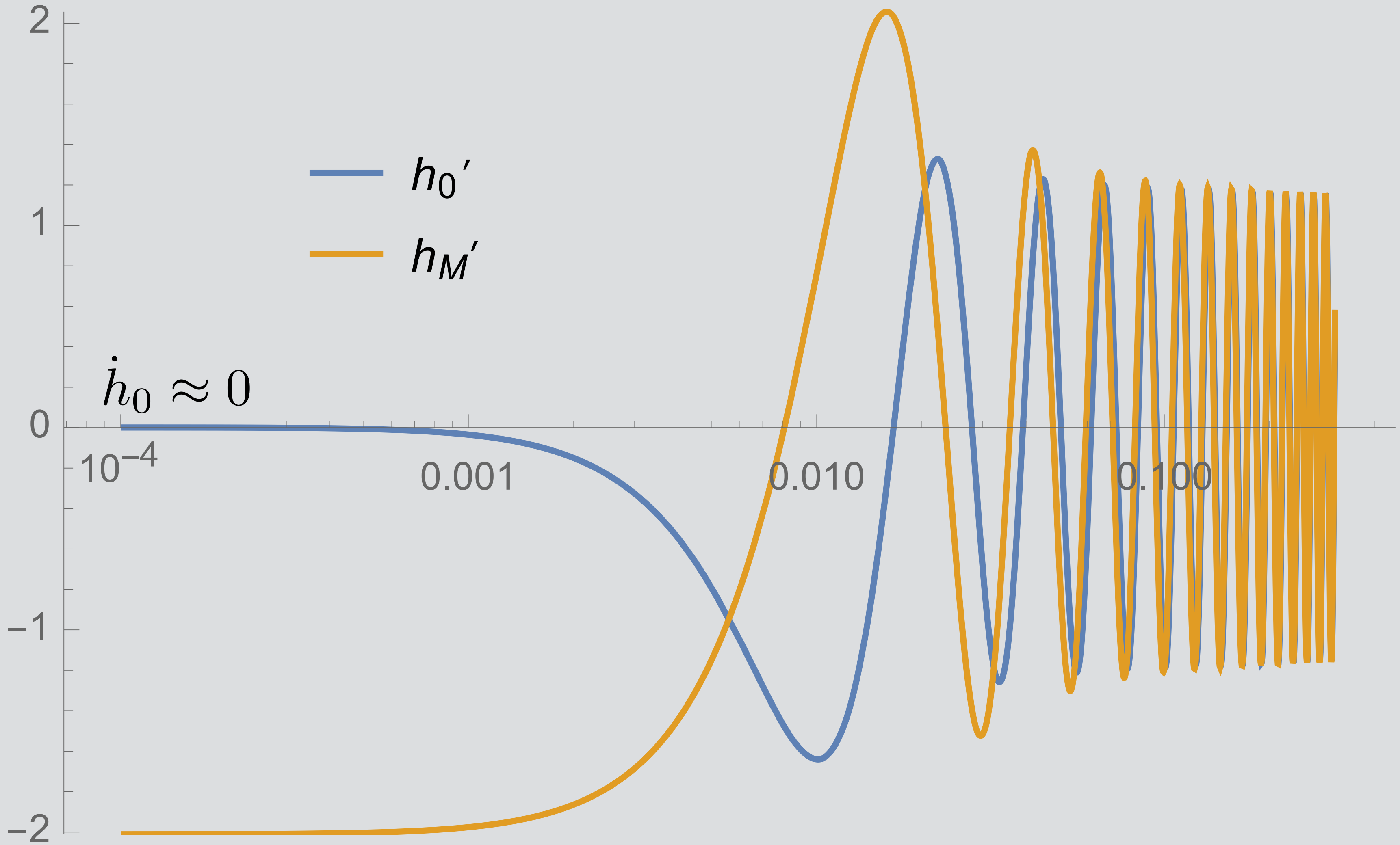
B modes

- At large scales source function,

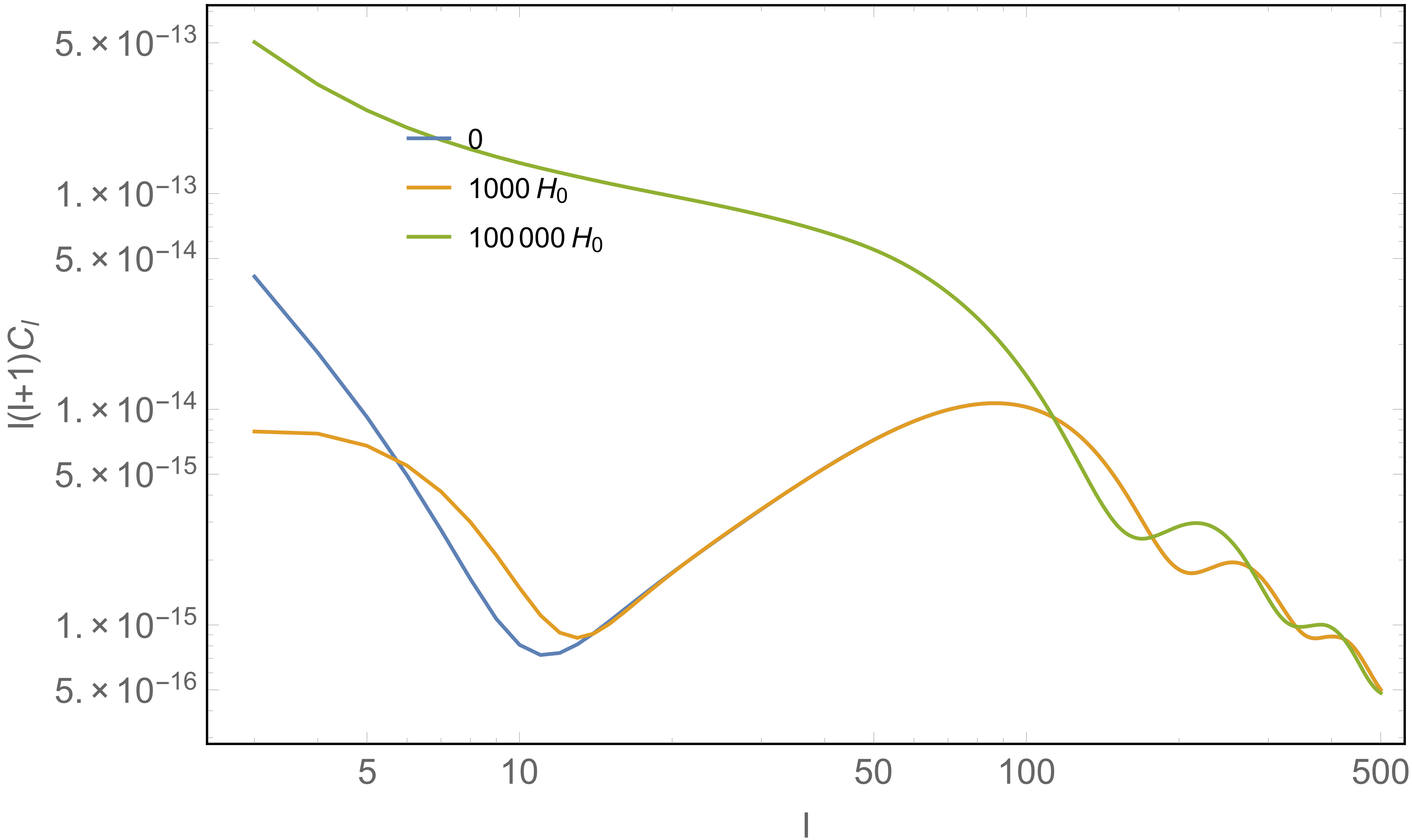
$$\Psi \propto \dot{h}_0 h_M + \dot{h}_M h_o$$

- If second term dominates then the B- modes are modified at large scales

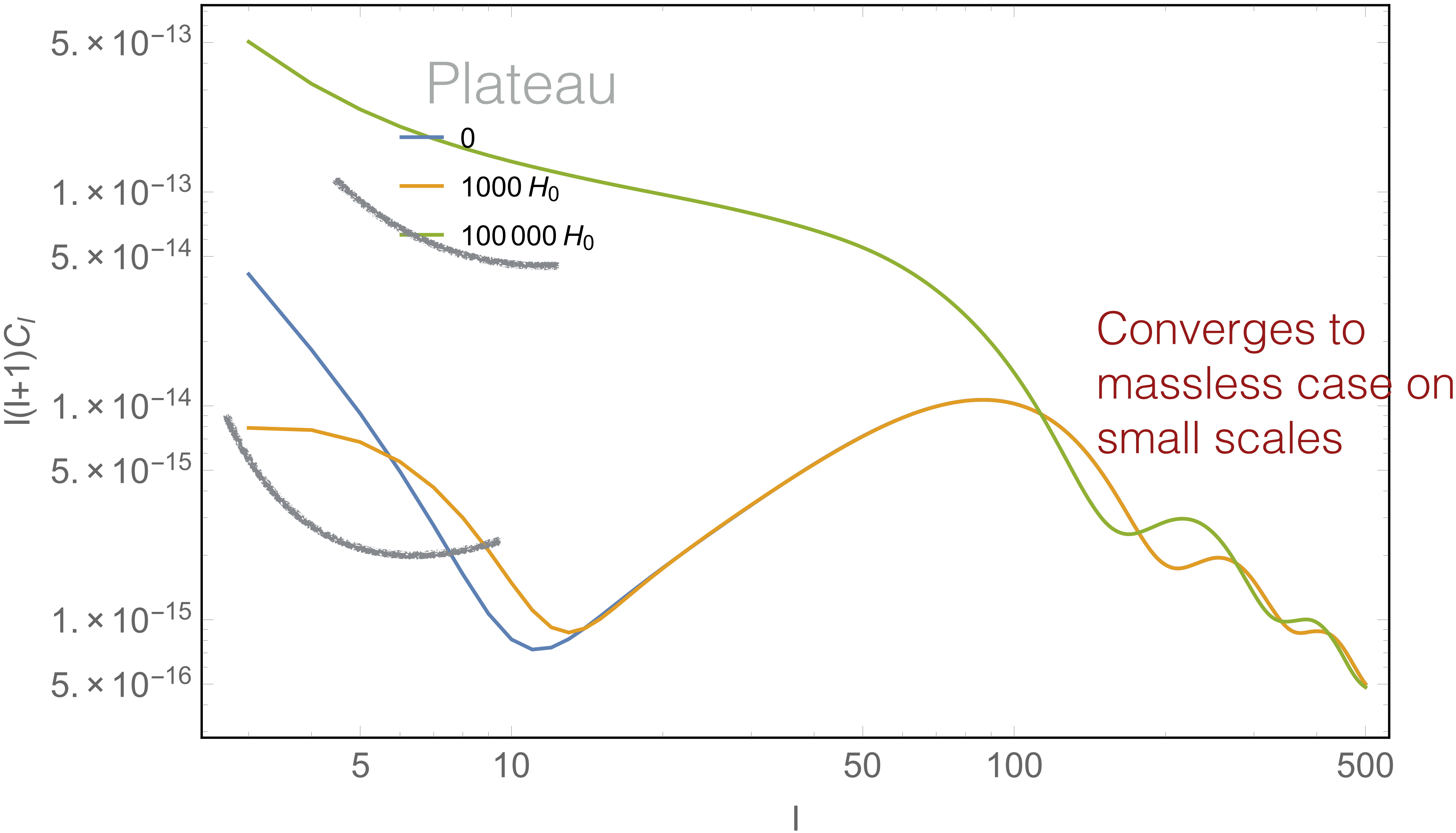
- One of the terms is zero while the other is not.



$$\dot{h}(\tau_{\rm rec}) \approx \frac{\cos(\frac{m\tau_{\rm rec}{}^3}{3})}{\tau_{\rm rec}}(1 - \frac{(k\tau_{\rm rec})^2}{10})$$



$5.\times10^{-13}$
$1.\times10^{-13}$
$5.\times10^{-14}$
$1.\times10^{-14}$
$5.\times10^{-15}$
$1.\times10^{-15}$
$5.\times10^{-16}$
$l(l+1)C_l$
5
10
50
100
500
l
0
$1000\,H_0$
$100\,000\,H_0$



Plateau
0
1000 H_0
100 000 H_0
Converges to massless case on small scales
$l(l+1)C_l$
$5.\times10^{-13}$
$1.\times10^{-13}$
$5.\times10^{-14}$
$1.\times10^{-14}$
$5.\times10^{-15}$
$1.\times10^{-15}$
$5.\times10^{-16}$
5
10
50
100
500
l

We can solve analytically in some limits

$$C_{ll}^{B} \approx P_h \frac{\cos^2(\frac{m{\tau_{\rm rec}}^3}{3})}{{\tau_{\rm rec}}^2} = \frac{P_h}{2{\tau_{\rm rec}}^2}\left(1+\cos\left(\frac{2m{\tau_{\rm rec}}^3}{3}\right)\right)$$

small l

Plateau

$$\frac{A_s r}{4\pi^{3/2}} \frac{g(\tau_r)\cos^2\left(\frac{m_g\tau_r^3}{3}\right)}{100\tau_r^2} e^{-\frac{D^2}{\Delta\tau_r^2}-\frac{l^2\Delta\tau_r^2}{D_r^2}} \delta^{(2)}(\boldsymbol{l}+\boldsymbol{l}')$$

large l

Decaying at large l

- We can only probe a range of masses where the second term dominates

- It is limited but still can give a very good bound on the graviton mass

- Because $H_{\rm rec} \simeq 3 \times 10^{-29} {\rm eV}$, a no detection will imply

$$\boxed{m_{\rm g} < 3 \times 10^{-29} {\rm eV}}$$

Flauger et al. (2009)

Bigravity

- Lorentz invariant massive gravity will require us to consider more than one graviton.
- What if we have more than one graviton, how much does the analysis we have change?.
- It is tricky as the effect might be smaller

Bigravity

- In general the theory has one light graviton coupled to the mass

- This means that the Boltzmann equation has to consider both fields. As we have described on large scales this implies that the signal is proportional to

$$\text{Signal} \propto \left[\kappa_1 * h_1 + \kappa_2 * h_2\right]^2$$

- In the case of bigravity

$$\delta S_{\mathrm{matter}} = \int \mathrm{d}^4 x \, \left\{ H^{+}_{ij} + \frac{\kappa \, \xi_c^2}{1 + \kappa \, \xi_c^2} H^{-}_{ij} \right\} T^{ij}$$

with $\kappa \ll \xi_c^{-2} \sim \mathrm{O}(1)$

Fassiello and Ribeiro (2015)

- This implies that the effect from the mass is suppressed for the B-modes

- Then is easier to look at the Bispectrum,

Doubly coupled

- Then it's obviously easier. One can have two modes and then if both are coupled the speed does not change but the overall effect is small

  Brax, Davis and Noller

- Conditions is on the source functions

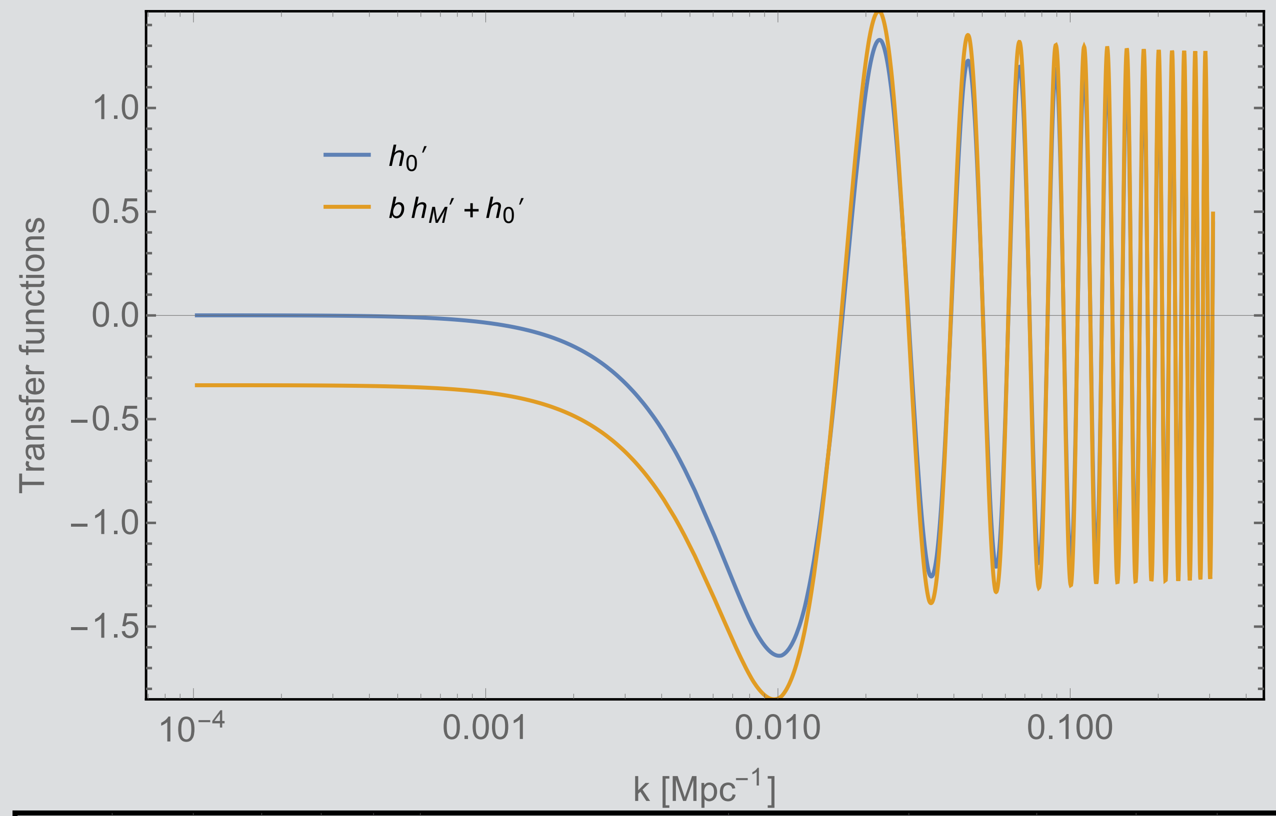
- The plateau does not disappear but is effect mildered

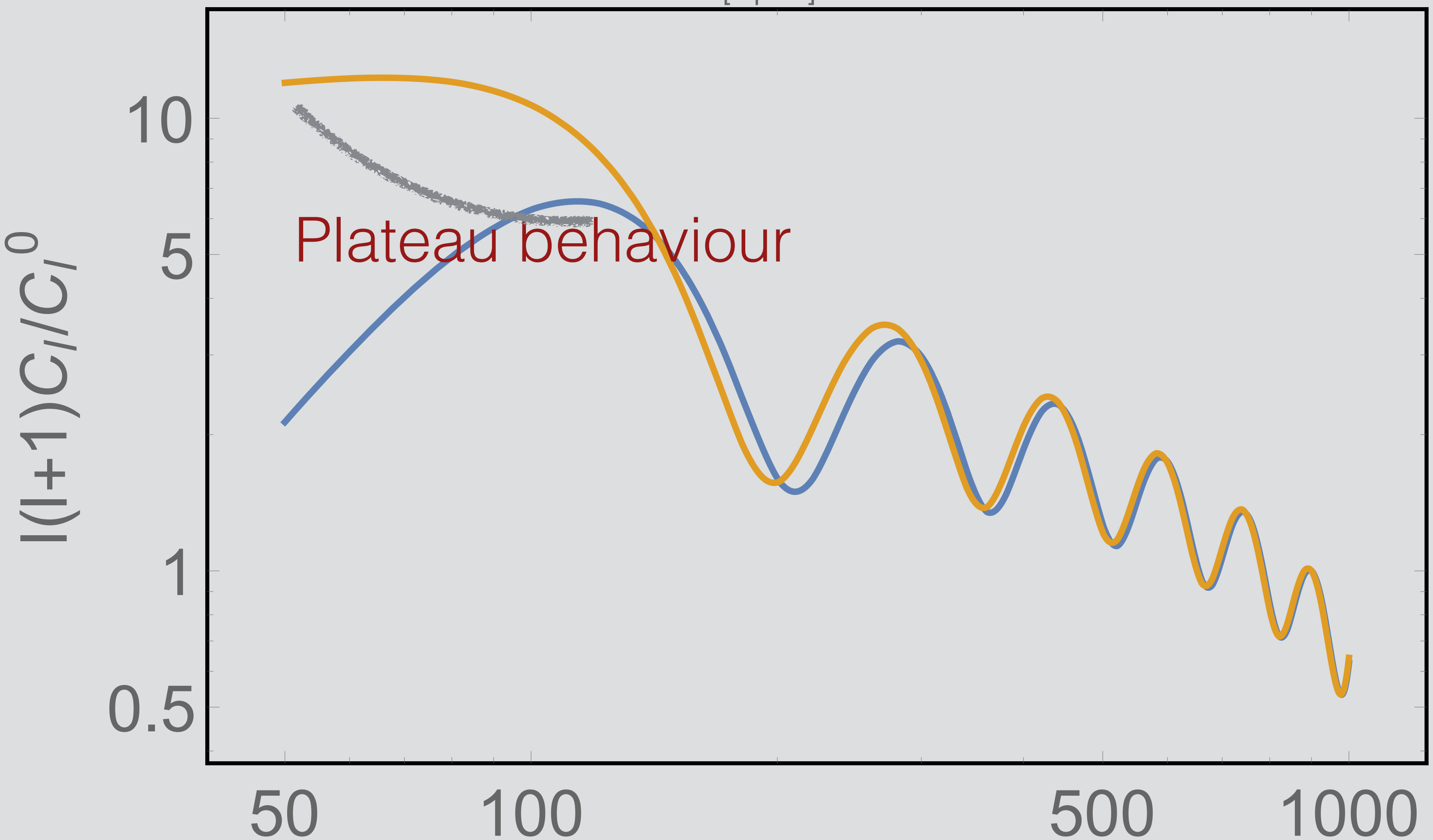
Bigravity

- The couplings changes, so the contribution to the source function can be larger
- Couplings now $\delta S_{\mathrm{matter}} = \int \mathrm{d}^4 x \left\{ \kappa_1 h^{(1)}_{ij} + \kappa_2 h^{(2)}_{ij} \right\} T^{ij}$
- We assume that the mass matrix is

$$\begin{pmatrix} M_{11}^2 a^2 & M_{21}^2 a^2 \\ M_{12}^2 a^2 & M_{22}^2 a^2 \end{pmatrix}$$

Doubly coupled

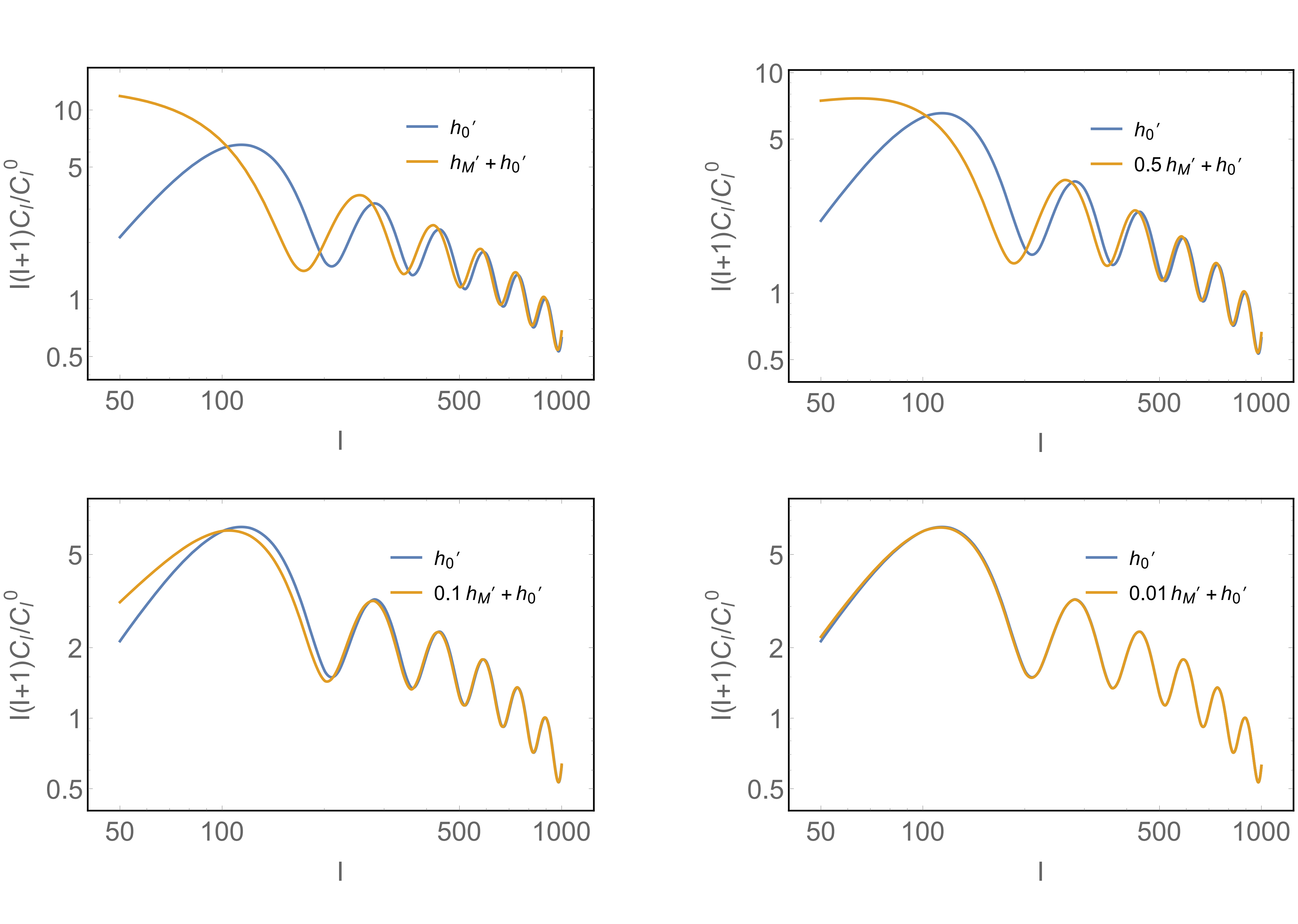




- We diagonalise the mass matrix, and assume for simplicity that its determinant its 0
- We get two gravitons with masses 0 and $\dfrac{M_{11}^2 + M_{22}^2}{2}$
- Coupling to matter is then

$$\delta S_{\rm matter} = M_{\rm Pl}^2 \int \mathrm{d}^4 x a_J \frac{a M_{12}^2}{2M_{12}^2 + M_{11}^2 - M_{22}^2} \left\{ (1 + M_{11}^2/M_{12}^2) f_{ij}^0 + (1 - M_{22}^2/M_{21}^2) f_{ij}^m \right\} T^{ij}$$

- In general both graviton will matter



h_0'
$h_M' + h_0'$
$0.5\, h_M' + h_0'$
$0.1\, h_M' + h_0'$
$0.01\, h_M' + h_0'$
$l(l+1)C_l/C_l^0$
10
5
2
1
0.5
50
100
500
1000
l

Instability

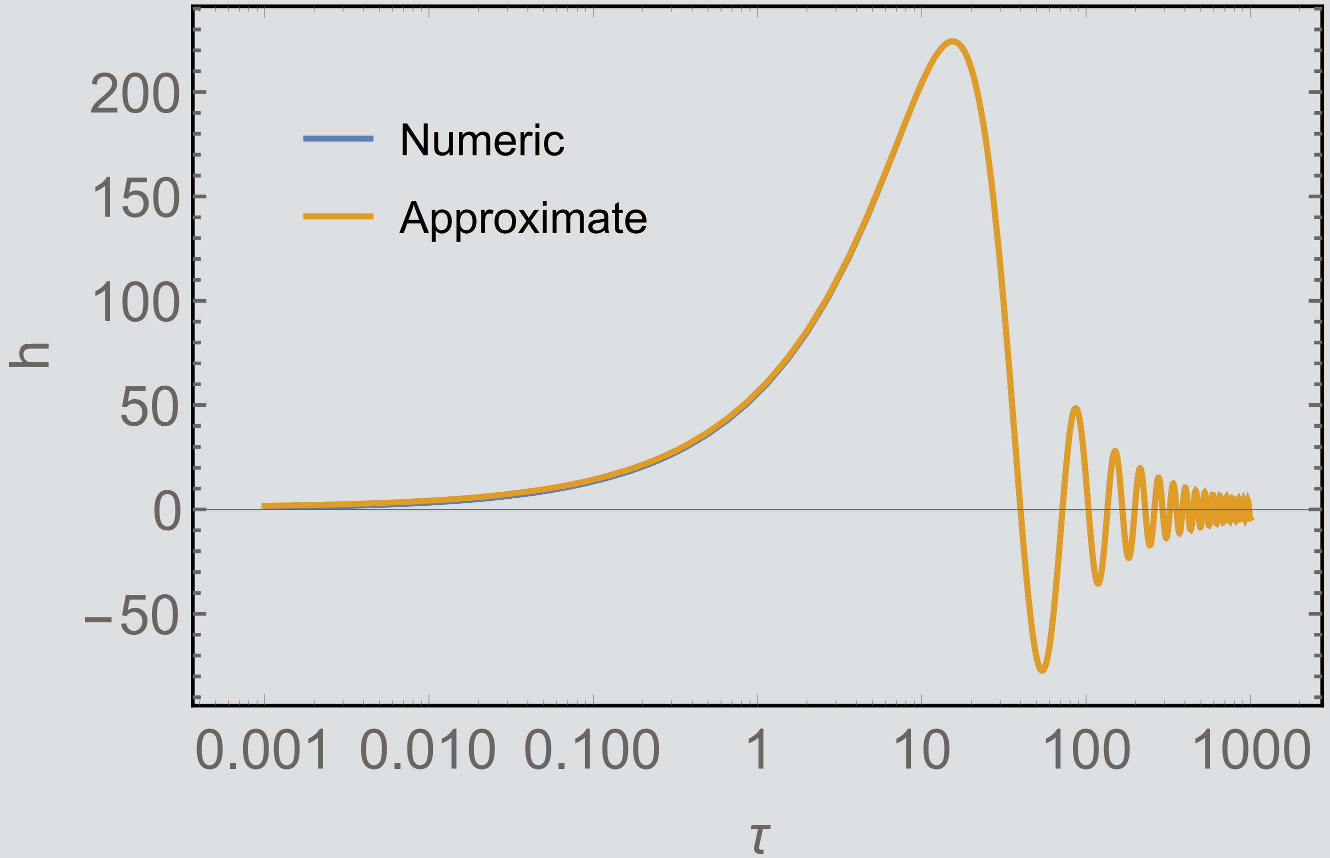
- The mass matrix is modified if we consider the coupling to pressure

$$\begin{pmatrix} M_{11}^2 a^2 - \frac{\beta 2^2}{(\beta 1^2 + \beta 2^2)} 3\omega (aH)^2 & M_{21}^2 a^2 + \frac{\beta 1 \beta 2}{(\beta 1^2 + \beta 2^2)} 3\omega (aH)^2 \\ M_{12}^2 a^2 + \frac{\beta 1 \beta 2}{(\beta 1^2 + \beta 2^2)} 3\omega (aH)^2 & M_{22}^2 a^2 - \frac{\beta 1^2}{(\beta 1^2 + \beta 2^2)} 3\omega (aH)^2 \end{pmatrix}$$

- Which introduces a mild instability during radiation domination.

- Considering a massive mode instable mode its solution is

$$h = \begin{cases} (mH_0\tau^2/2)^{1/4} j_{-1/4}(mH_0\tau^2/2) \left(j_0(k\tau) + j_{1/2(-1+\sqrt{5}))}(k\tau) \right) & \tau < \tau_{eq} \\ 3\frac{1}{k\tau} \times j_0(\frac{1}{3} m H_0^2 \tau^3) \left(A j_1(k\tau) + B y_1(k\tau) \right) & \tau > \tau_{eq} \end{cases}$$

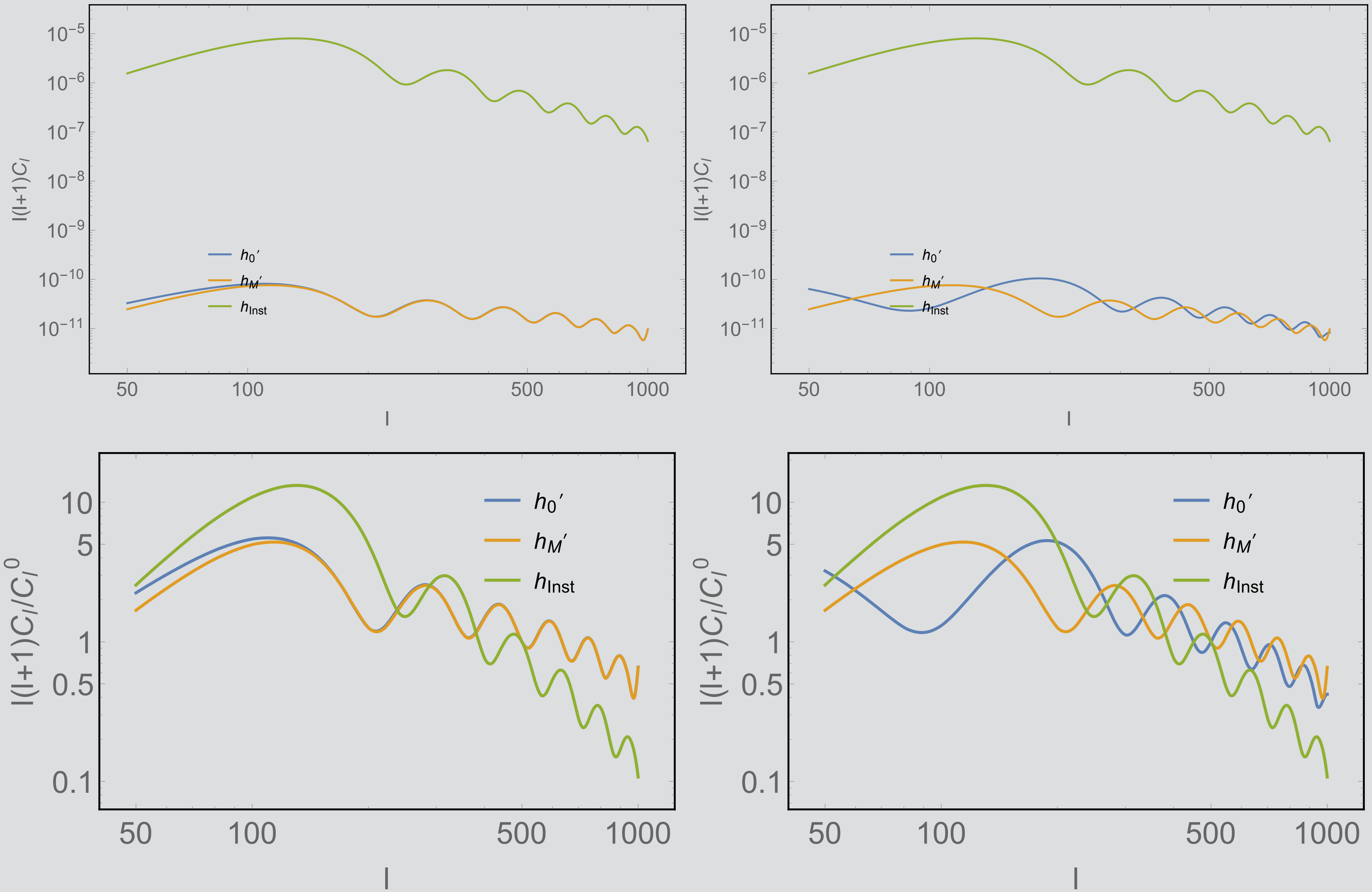


Coupling to matter

- We can solve the equations for the mass matrix in the case that,
- We get a massless mode unstable at radiation domination and a stable massive mode

$$\delta S_{\rm matter} = M_{\rm Pl}^2 \int \mathrm{d}^4 x a_J \frac{a}{2M_{12}^2 + M_{11}^2 + M_{22}^2 + \frac{(\beta_1-\beta_2)^2}{(\beta_1^2+\beta_2^2)\tau^4}} \times \left\{ (M_{12}^2 + M_{11}^2 + \frac{\beta_1^2 + \beta_1\beta_2}{\beta_1^2 + \beta_2^2}\frac{1}{\tau^4}) f_{ij}^0 + (M_{21}^2 - M_{22}^2 + \frac{\beta_1\beta_2 - \beta_2^2}{\beta_1^2+\beta_2^2}\frac{1}{\tau^4}) f_{ij}^m \right\} T^{ij}$$

Always positive



10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
$l(l+1)C_l$
h_0'
h_M'
$h_{\rm Inst}$
50
100
500
1000
l
10
5
1
0.5
0.1
$l(l+1)C_l/C_l^0$

- Instability increases the amplitude of the power spectrum.
- It cannot be removed by fine-tuning the mass parameters
- if r is very low the signal could still be within the current limits

Conclusions

- If B-modes are detected we can learn a lot about gravity as well.
- It will impose good bounds if we allow the graviton to vary its mass and speed
- We can also look at the bispectrum to improve our constraints